TRINITY COLLEGE

Semester Two Examination, 2020

Question/Answer booklet

MATHEMATICS SPECIALIST UNITS 3&4

Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable): | |

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

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Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(6 marks)**

The time interval T between vehicles arriving at a 24-hour service station is known to follow an exponential distribution with a standard deviation of 42 seconds.

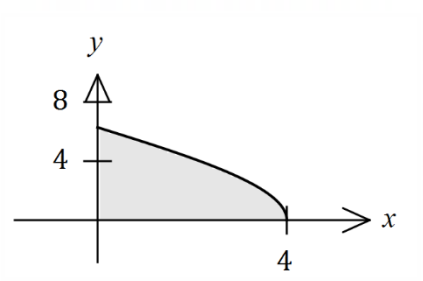
The mean of a random sample of 80 time intervals was 45 seconds.

(a) Use the sample to construct a 90% confidence interval for the mean of T. (4 marks)

(b) State the key assumption made when constructing the interval in part (a) and explain how confident you are that the assumption is valid. (2 marks)

Question 10 **(5 marks)**

The graph of $\frac{x}{4} = \cos\left(\frac{y}{4}\right)$ is shown for $0 \leq y \leq 2\pi$.



Show that when the shaded region bounded by the curve and the x-axis is rotated about the y-axis, the volume of revolution of the solid formed is $16\pi^2$.

Question 11 **(7 marks)**

The growth rate of electric vehicle (EV) sales as a percentage P of all passenger vehicle (PV) sales in Australia can be modelled by

$$\frac{dP}{dt} = rP(k - P)$$

At the start of 2013 ($t = 0$ years), EV sales were 0.03% of all PV sales in Australia. 4 years later, P had increased from 0.03% to $P = 0.21\%$. The maximum expected percentage of EV sales is 65%.

(a) Using a standard formula, or otherwise, show that $P \approx \frac{65}{1+2165.67e^{-0.4827t}}$. (3 marks)

(b) Determine

(i) the percentage of EV sales expected at the start of 2025. (1 mark)

(ii) the year in which EV sales are expected to reach 50% of all PV sales. (1 mark)

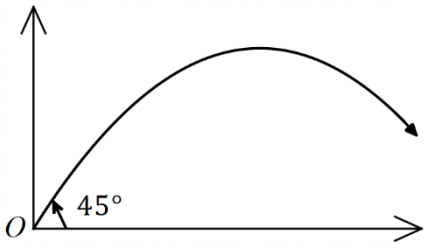
(c) State the year in which the growth rate of EV sales as a percentage of PV sales will reach a maximum and determine this maximum growth rate. (2 marks)

See next page

Question 12 **(6 marks)**

A small projectile is launched upwards from O at an angle of $45°$ to the horizontal, with an initial speed of 24 m/s.

The motion of the projectile is only affected by gravity, so that the acceleration at any time t seconds is given by $\mathbf{a}(t) = -9.8\mathbf{j}$ m/s^2.



(a) Show that the position vector of the projectile relative to O after t seconds is given by $\mathbf{r}(t) = \left(12\sqrt{2}t\right)\mathbf{i} + \left(12\sqrt{2}t - 4.9t^2\right)\mathbf{j}$ m. (3 marks)

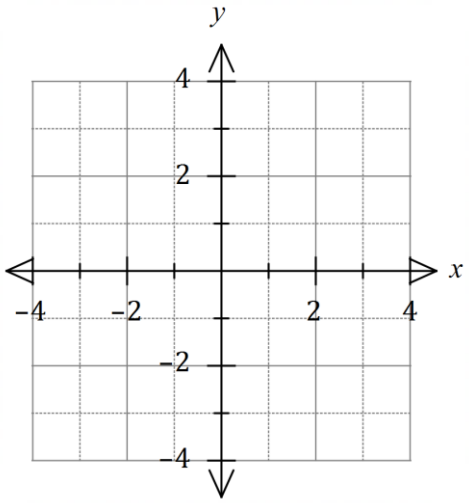
(b) Determine the maximum altitude of the projectile above O and the time taken to reach this altitude. (3 marks)

See next page

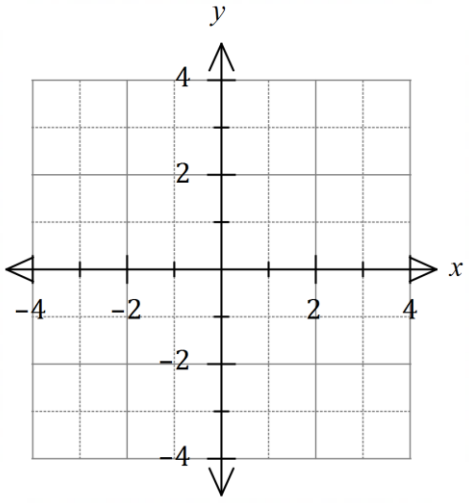
Question 13 **(9 marks)**

(a) Sketch the locus of the complex number $z = x + iy$ given by

(i) $|z + 2 - 3i| > |z - 2 + i|$. (3 marks)



(ii) $|z - 1 + 2i| \leq 2$. (3 marks)



(b) For the locus $|z - 1 + 2i| \leq 2$ determine, correct to the nearest degree, the minimum value of $\arg(z)$, $-180° < \arg(z) \leq 180°$. (3 marks)

See next page

Question 14 **(10 marks)**

The shell weight W of eggs laid by hens in a flock is known to be normally distributed with mean of 6.54 g and standard deviation 0.56 g.

(a) A random sample of 40 eggs is selected from the flock and the mean shell weight of these eggs calculated.

(i) State the distribution of $\overline{W}$, the sample mean. (3 marks)

(ii) Determine the probability that the sample mean is between 6.5 g and 6.6 g. (2 marks)

(iii) Suppose the size of the random sample was halved. Explain, without any further calculation, how this will affect your answer to part (a)(ii). (2 marks)

(b) Random samples of n eggs were repeatedly selected from the flock and the mean weight of each sample recorded. It was observed that 8% of the sample means weighed less than 6.44 g. Determine the value of n. (3 marks)

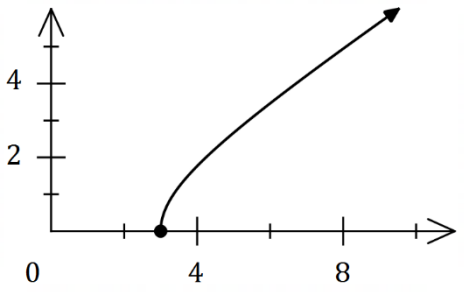
See next page

Question 15 **(8 marks)**

The path of a particle is shown in the diagram.

Its position, in metres, relative to the origin O at time t seconds is given by

$\mathbf{r}(t) = 3\sec(t)\,\mathbf{i} + 2\tan(t)\,\mathbf{j},\ 0 \leq t < \frac{\pi}{2}$.



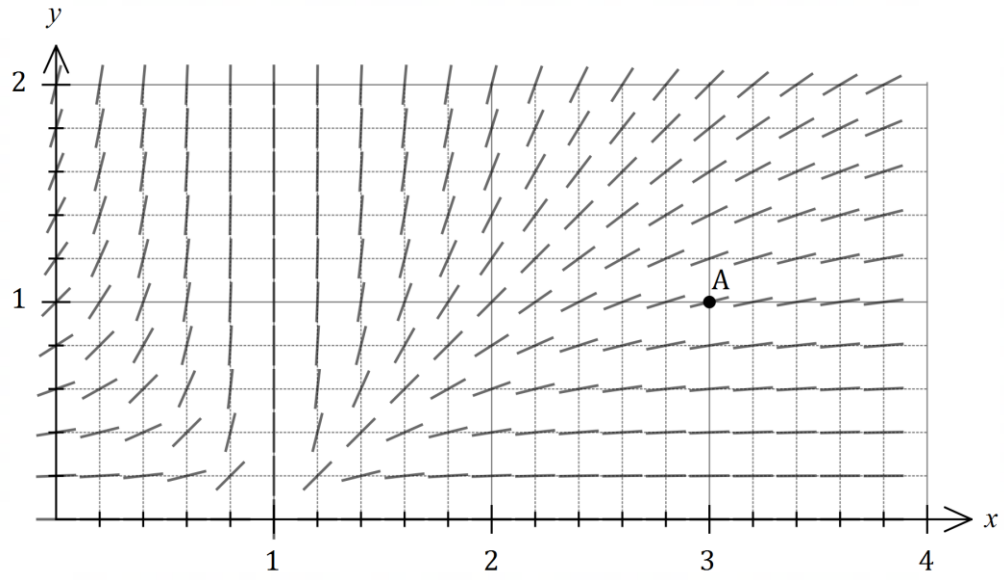
(a) Determine the Cartesian equation of the path of the particle. (3 marks)

(b) Determine the exact speed of the particle when $t = \frac{\pi}{3}$. (3 marks)

(c) Determine, correct to two decimal places, the distance the particle travels between $t = 0$ and $t = \frac{\pi}{3}$. (2 marks)

Question 16 **(6 marks)**

The graph below shows the slope field for the differential equation $\frac{dy}{dx} = \frac{y^2}{(x-1)^2}$.



(a) Determine the value of the slope field at point A. (1 mark)

(b) On the axes above, sketch the solution curve for the differential equation that passes through point A. (1 mark)

(c) Determine the particular solution $y = f(x)$ to the differential equation that has initial solution $f(3) = 1$. (4 marks)

See next page

Question 17 **(10 marks)**

Lines L_1 and L_2 have equations $\mathbf{r} = \begin{pmatrix} 14 \\ -6 \\ -8 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \\ -3 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 6 \\ -7 \\ -5 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix}$ respectively, they both lie in plane Π and they intersect at point P.

(a) Determine coordinates of point P. (3 marks)

(b) Determine the Cartesian equation of plane Π. (4 marks)

Sphere S has a radius of 12, is tangential to plane Π at point P and the origin lies within it.

(c) Determine the vector equation of sphere S. (3 marks)

See next page

Question 18 **(8 marks)**

Let $u = -2 + \sqrt{12}i$ and $v = 1 + i$.

(a) Solve the equation $z^3 = 2u$, giving all solutions in the form $r\,\text{cis}\,\theta$, $-\pi < \theta \leq \pi$. (3 marks)

(b) Express w in both Cartesian and polar forms, where $w = u \div v$. (2 marks)

(c) Show how to use your answers from part (b) to determine an exact value for

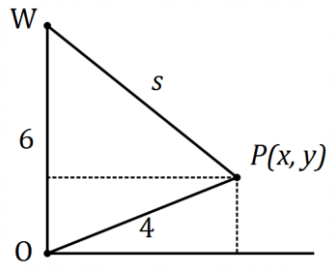
(i) $\sin\left(\frac{5\pi}{12}\right)$. (2 marks)

(ii) $\tan\left(\frac{5\pi}{12}\right)$. (1 mark)

See next page

Question 19 **(7 marks)**

A light rope from winch W, at the top of a six-metre-tall wall OW, is attached to point P at the end of a four-metre-long pole OP.



The winch is winding the rope in at a rate of 15 cm/s so that the pole is rotating about O from a horizontal to a vertical position.

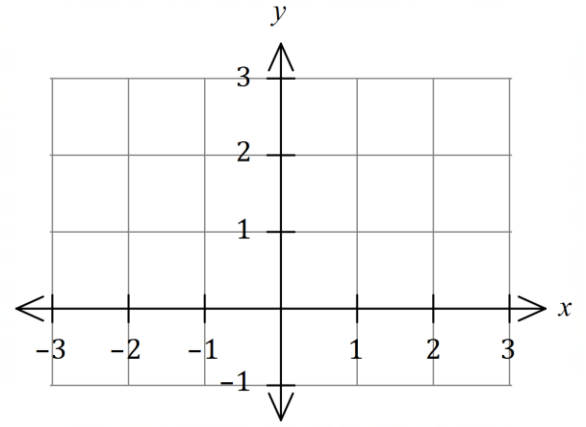
Let s be the length of the rope PW, and let x be the horizontal distance and y be the vertical distance of P relative to O.

Show that $s^2 = 52 - 12y$ and hence determine the rate at which x is decreasing at the instant that y is increasing at a rate of 10 cm/s.

Question 20 **(8 marks)**

Consider the function $f(z) = -i\bar{z} + 1 + i$, where $z = x + iy$ and $x, y \in \mathbb{R}$.

(a) Determine $f(z_1)$ when $z_1 = 1 + 3i$ and use the Argand diagram below to show that $f(z_1)$ is a reflection of z_1 in the line $x + y = 1$. (3 marks)



Any reflection of z in the complex plane can be expressed in the form $g(z) = a\bar{z} + b$, where a and b are complex constants.

(b) By considering the transformations of the axes intercepts of the reflection line, or otherwise, determine the value of a and the value of b so that $g(z)$ represents a reflection of z in the line $y = 3x - 3$. (4 marks)

(c) Given $w = 20 - 18i$, determine w', the reflection of w in the line $y = 3x - 3$. (1 mark)

See next page

Question 21 **(8 marks)**

Particles P and Q travel in a straight line with displacement x m and velocity v m/s at time t s.

(a) The acceleration of P is given by $a = -\frac{v}{5}$ m/s^2, and when $t = 0$, $x = 0$ and $v = 24$. Determine, correct to one decimal place, the displacement of P after 4 seconds. (5 marks)

(b) The acceleration of Q is given by $a = -1 + \sqrt{v^2 + 3}$. Determine, correct to two decimal places, the time taken for its velocity to increase from 2 m/s to 15 m/s. (3 marks)

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____